**Computational Astrophysics HW1**

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* **Derive the stability criterion of the Lax-Wendroff scheme for solving the advection equation; demonstrate that it is second-order accurate.**

1. Stability Criterion of the Lax-Wendroff scheme

Step1:

Step2:

Let, and bring step1 inside to step2, we have

Insert ,

So

1. Second order accurate]

From step2, we get

On LHS, write and , and do the Taylor expansion with the center .

Same method and argument goes with expanding the RHS. Write and , expand with the center . We have,

We can see that Lax-Wendroff Scheme is 2nd order accurate.

By testing with the code directly, we can also see that it is 2nd order accurate. If there are 2 times of the previous sample points, is cut to half, then the error decreases by a factor of 4.

|  |  |
| --- | --- |
| N sample points | Error |
| 100 |  |
| 200 |  |
| 400 |  |
| 800 |  |

* **Demonstrate that the Crank-Nicolson scheme is unconditionally stable for solving the diffusion equation.**

Crank-Nicolson Scheme:

Let , and insert ,

Since ,

will always satisfy. So Crank Nicolson Scheme is unconditionally stable.